CBSE Class 9 Mathemaics

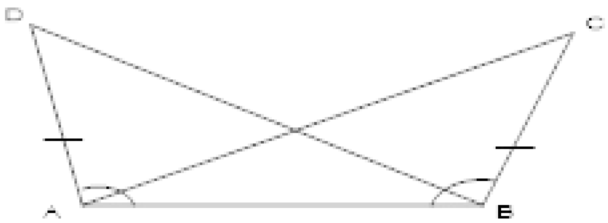
Important Questions

Chapter 7

Triangles

1 Marks Quetions

1. In fig, if AD =BC and ∠ BAD = ∠ ABC, then ∠ ACB is equal to



(A) ∠ ABD

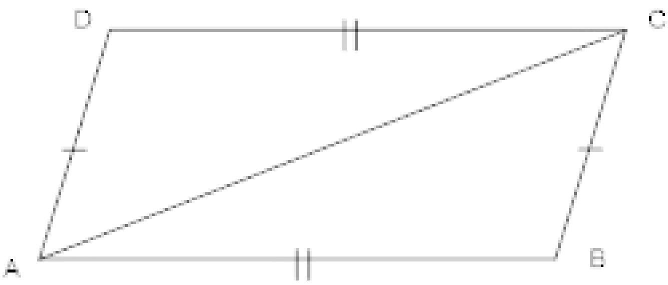
(B) ∠ BAD

(C) ∠ BAC

(D) ∠ BDA

Ans. (D) ∠ BDA

2. IN fig, if ABCD is a quadrilateral in which AD= CB, AB=CD, and ∠ D= ∠ B, then ∠ CAB is equal to



(A) ∠ ACD

(B) ∠ CAD

(C) $\angle$ **ACD**

(D) $\angle$ **BAD**

Ans. (C) $\angle$ ACD

3. If O is the mid – point of AB and $\angle$ **BQO =** $\angle$ **APO, then** $\angle$ **OAP is equal to**

(A) $\angle$ **QPA**

(B) $\angle$ **OQB**

(C) $\angle$ **QBO**

(D) $\angle$ **BOQ**

Ans. (C) $\angle$ QBO

4. IF AB $\perp$ **BC and** $\angle$ **A =** $\angle$ **c, then the true statement is**

(A) AB $\neq$ **AC**

(B) AB=BC

(C) AB=AD

(D) AB=AC

Ans. (B) AB=BC

5. If $\triangle$ **ABC is an isosceles triangle and** $\angle$ **B =** 65^0**, find x.**

(a) 60^0

(b) 70^0

(c) 50^0

(d) none of these

Ans. (c) 50^0

6. If AB=AC and $\angle$ ACD= 120^0, find $\angle$ A

(a) 50^0

(b) 60^0

(c) 70^0

(d) none of these

Ans. (b) 60^0

7. What is the sum of the angles of a quadrilateral:

(a) 260^0

(b) 360^0

(c) 180^0

(d) 90^0

Ans. (b) 360^0

8. The sum of the angles of a triangle will be:

(a) 360^0

(b) 270^0

(c) 180^0

(d) 90^0

Ans. (c) 180^0

9. An angle is 14° more than its complement. Find its measure.

(A) 42

(B) 32

(C) 52

(D) 62

Ans. (C) 52

10. An angle is 4 time its complement. Find measure.

(A) 62

(B) 72

(C) 52

(D) 42

Ans. (B) 72

11. Find the measure of angles which is equal to its supplementary.

(A) 120°

(B) 60°

(C) 45°

(D) 90°

Ans. (D) 90°

12. Which of the following pairs of angle are supplementary?

(A) $30^\circ, 120^\circ$

(B) $45^\circ, 135^\circ$

(C) $120^\circ, 30^\circ$

(D) None of these.

Ans. (B) $45^\circ, 135^\circ$

13. Find the measure of each exterior angle of an equilateral triangle.

(A) 110°

(B) 100°

(C) 120°

(D) 150°

Ans. (C) 120°

14. In an isosceles ΔABC, if AB=AC and $\angle A = 90^\circ$, Find $\angle$B.

(A) 45°

(B) 80°

(C) 95°

(D) 60°

Ans. (A) 45°

15. In a ΔABC, if $\angle$B= $\angle$C= 45°, Which is the longest side.

(A) BC

(B) AC

(C) CA

(D) None of these.

Ans. (A) BC

16. In a $\triangle$ABC, if AB=AC and $\angle$ B=70° , Find $\angle$ A.

(A) 40°

(B) 50°

(C) 45°

(D) 60°

Ans. (A) 40°

17. *In* a Δ ABC, If $\angle A = 45^0$ and $\angle B = 70^0$. **Determine the shortest sides of the triangles.**

(a) AC

(b) BC

(c) CA

(d) none of these

Ans. (b) BC

18. In an $\triangle$ABC, if $\angle A$ = 45^0 and $\angle B$ = 70^0, determine the longest sides of the triangle.

(a) AC

(b) CA

(c) BC

(d) none of these

Ans. (a) AC

19. The sum of two angles of a triangle is equal to its third angle. Find the third angles.

(a) 90^0

(b) 45^0

(c) 60^0

(d) 70^0

Ans. (a) 90^0

20. Two angles of triangles are $65^0\ and\ 45^0$ **respectively. Find third angles.**

(a) 90^0

(b) 45^0

(c) 60^0

(d) 70^0

Ans. (d) 70^0

21. Δ **ABC is an isosceles triangle with AB=AC and** $\angle B = 45^0$**, find** $\angle A$ **.**

Ans. $In\ \Delta\ ABC,$

$AB = AC$

$\Rightarrow \angle B = \angle C$ [angle opposite to equal sides are equal]

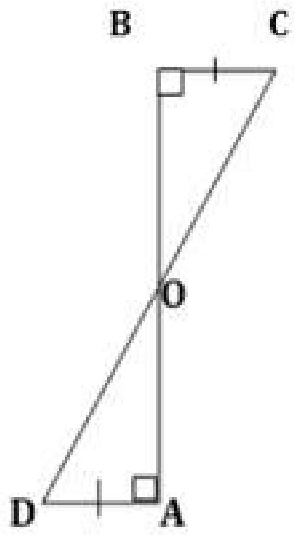
But, $\angle B = 45^0 = \angle C$

And, $\angle A + \angle B + \angle C = 180^0$

$\angle A + 90^0 = 180^0$

$\angle A = 90^0$

22. 1. AD and BC are equal perpendiculars to a line segment AB. Show that CD bisects AB (See figure)



Ans. In $\triangle$BOC and $\triangle$AOD,

$\angle$OBC = $\angle$OAD = 90° [Given]

$\angle$BOC = $\angle$AOD [Vertically Opposite angles]

BC = AD [Given]

$\therefore$ $\triangle$BOC $\cong$ $\triangle$AOD [By ASA congruency]

$\Rightarrow$ OB = OA and OC = OD [By C.P.C.T.]

CBSE Class 9 Mathemaics

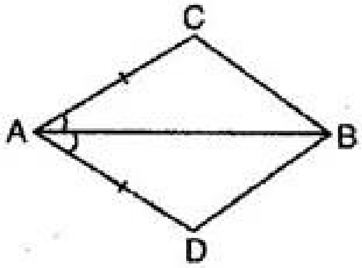
Important Questions

Chapter 7

Triangles

2 Marks Quetions

1. In quadrilateral ABCD (See figure). AC = AD and AB bisects ∠A. Show that △ABC ≅ △ABD. What can you say about BC and BD?



Ans. Given: In quadrilateral ABCD, AC = AD and AB bisects ∠A.

To prove: △ABC ≅ △ABD

Proof: In △ABC and △ABD,

AC = AD [Given]

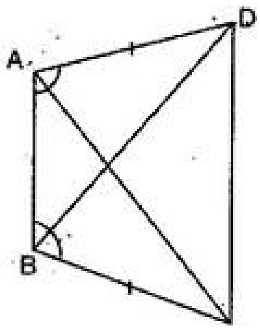
∠BAC = ∠BAD [∵ AB bisects ∠A]

AB = AB [Common]

∴ △ABC ≅ △ABD [By SAS congruency]

Thus BC = BD [By C.P.C.T.]

2. ABCD is a quadrilateral in which AD = BC and ∠DAB = ∠CBA. (See figure). Prove that:



(i) △ABD ≅ △BAC

(ii) BD = AC

(iii) $\angle$ **ABD =** $\angle$ **BAC**

Ans. (i) In $\triangle$ABC and $\triangle$ABD,

BC = AD [Given]

$\angle$ DAB = $\angle$ CBA [Given]

AB = AB [Common]

$\therefore$ $\triangle$ABC $\cong$ $\triangle$ ABD [By SAS congruency]

Thus AC = BD [By C.P.C.T.]

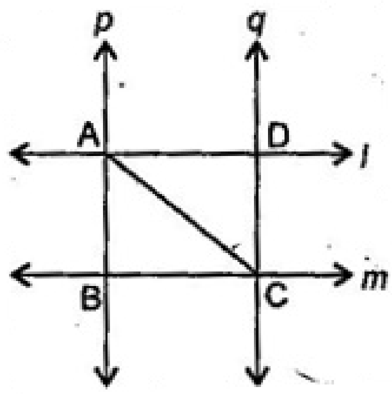
(ii) Since $\triangle$ABC $\cong$ $\triangle$ ABD

$\therefore$ AC = BD [By C.P.C.T.]

(iii) Since $\triangle$ABC $\cong$ $\triangle$ABD

$\therefore$ $\angle$ ABD = $\angle$ BAC [By C.P.C.T.]

3. l **and** m **are two parallel lines intersected by another pair of parallel lines** p **and** q **(See figure). Show that** $\triangle$**ABC** $\cong$ $\triangle$ **CDA.**



Ans. AC being a transversal. [Given]

Therefore $\angle$ DAC = $\angle$ ACB [Alternate angles]

Now $p \parallel q$ [Given]

And AC being a transversal. [Given]

Therefore $\angle$BAC = $\angle$ACD [Alternate angles]

Now In $\triangle$ABC and $\triangle$ADC,

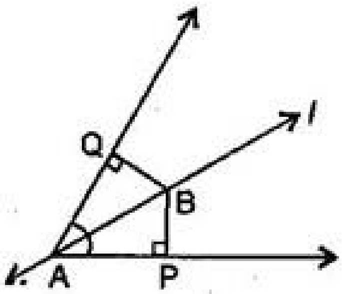
$\angle$ACB = $\angle$DAC [Proved above]

$\angle$BAC = $\angle$ACD [Proved above]

AC = AC [Common]

$\therefore$ $\triangle$ABC $\cong$ $\triangle$CDA [By ASA congruency]

4. Line l is the bisector of the angle A and B is any point on l. BP and BQ are perpendiculars from B to the arms of $\angle$A. Show that:



(i) $\triangle$APB $\cong$ $\triangle$AQB

(ii) BP = BQ or P is equidistant from the arms of $\angle$A (See figure).

Ans. Given: Line l bisects $\angle$A.

$\therefore$ $\angle$BAP = $\angle$BAQ

(i) In $\triangle$ABP and $\triangle$ABQ,

$\angle$BAP = $\angle$BAQ [Given]

$\angle$BPA = $\angle$BQA = 90° [Given]

AB = AB [Common]

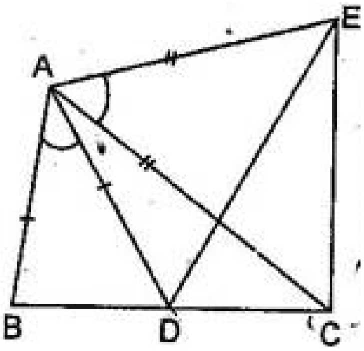
$\therefore$ $\triangle$APB $\cong$ $\triangle$AQB [By ASA congruency]

(ii) Since $\triangle$APB $\cong$ $\triangle$AQB

∴ BP = BQ [By C.P.C.T.]

⟹ B is equidistant from the arms of ∠A.

5. In figure, AC = AB, AB = AD and ∠BAD = ∠EAC. Show that BC = DE.



Ans. Given that ∠BAD = ∠EAC

Adding ∠DAC on both sides, we get

∠BAD + ∠DAC = ∠EAC + ∠DAC

⟹ ∠BAC = ∠EAD ....(i)

Now in △ABC and △AED,

AB = AD [Given]

AC = AE [Given]

∠BAC = ∠DAE [From eq. (i)]

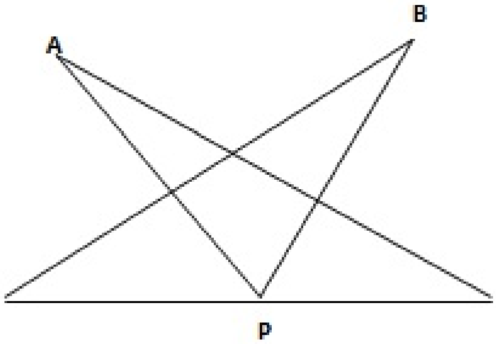
∴ △ABC ≅ △ADE [By SAS congruency]

⟹ BC = DE [By C.P.C.T.]

6. AB is a line segment and P is the mid-point. D and E are points on the same side of AB such that ∠BAD = ∠ABE and ∠EPA = ∠DPB. Show that:

(i) △DAF ≅ △FBPE D

(ii) AD = BE (See figure)



Ans. Given that ∠EPA = ∠DPB

Adding ∠EPD on both sides, we get

∠EPA + ∠EPD = ∠DPB + ∠EPD

⇒ ∠APD = ∠BPE ....(i)

Now in △APD and △BPE,

∠PAD = ∠PBE [∵ ∠BAD = ∠ABE (given), ∴ ∠PAD = ∠PBE]

AP = PB [P is the mid-point of AB]

∠APD = ∠BPE [From eq. (i)]

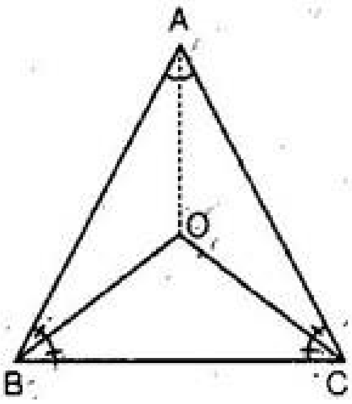
∴ △DPA ≅ △EBP [By ASA congruency]

⇒ AD = BE [ByC.P.C.T.]

7. In an isosceles triangle ABC, with AB = AC, the bisectors of ∠B and ∠C intersect each other at O. Join A to O. Show that:

(i) OB = OC

(ii) AO bisects ∠A.



Ans. (i) ABC is an isosceles triangle in which AB = AC.

∴ ∠C = ∠B [Angles opposite to equal sides]

$\Rightarrow \angle OCA + \angle OCB = \angle OBA + \angle OBC$

$\because$ OB bisects $\angle B$ and OC bisects $\angle C$

$\therefore \angle OBA = \angle OBC$ and $\angle OCA = \angle OCB$

$\Rightarrow \angle OCB + \angle OCB = \angle OBC + \angle OBC$

$\Rightarrow 2\angle OCB = 2\angle OBC$

$\Rightarrow \angle OCB = \angle OBC$

Now in $\triangle$OBC,

$\angle OCB = \angle OBC$ [Prove above]

$\therefore$ OB = OC [Sides opposite to equal sides]

(ii) In $\triangle$AOB and $\triangle$AOC,

AB = AC [Given]

$\angle OBA = \angle OCA$ [Given]

And $\angle B = \angle C$

$\Rightarrow \frac{1}{2}\angle B = \frac{1}{2}\angle C$

$\Rightarrow \angle OBA = \angle OCA$

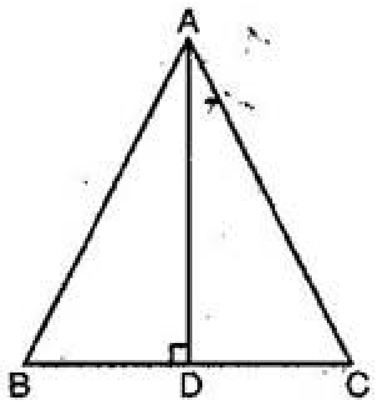
$\Rightarrow$ OB = OC [Prove above]

$\therefore \triangle AOB \cong \triangle AOC$ [By SAS congruency]

$\Rightarrow \angle OAB = \angle OAC$ [By C.P.C.T.]

Hence AO bisects $\angle A$.

8. In $\triangle$ABC, AD is the perpendicular bisector of BC (See figure). Show that $\triangle$ABC is an isosceles triangle in which AB = AC.



Ans. In $\triangle$AOB and $\triangle$AOC,

BD = CD [AD bisects BC]

$\angle$ADB = $\angle$ADC = 90° [AD $\perp$ BC]

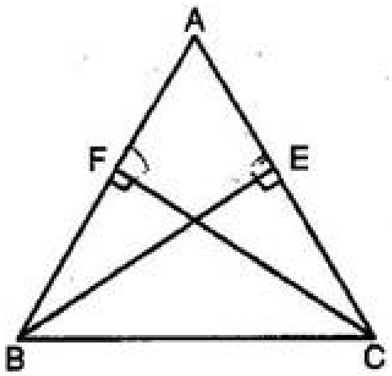
AD = AD [Common]

$\therefore$ $\triangle$ABD $\cong$ $\triangle$ACD [By SAS congruency]

$\Rightarrow$ AB = AC [By C.P.C.T.]

Therefore, ABC is an isosceles triangle.

9. ABC is an isosceles triangle in which altitudes BE and CF are drawn to sides AC and AB respectively (See figure). Show that these altitudes are equal.



Ans. In $\triangle$ABE and $\triangle$ACF,

$\angle$A= $\angle$A [Common]

$\angle$AEB = $\angle$AFC = 90° [Given]

AB = AC [Given]

$\therefore$ $\triangle$ABE $\cong$ $\triangle$ ACF [By ASA congruency]

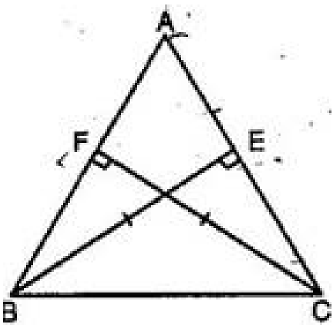
$\Rightarrow$ BE = CF [By C.P.C.T.]

$\Rightarrow$ Altitudes are equal.

10. ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal (See figure). Show that:

(i) $\triangle$ ABE $\cong$ $\triangle$ ACF

(ii) AB = AC or $\triangle$ ABC is an isosceles triangle.



Ans. (i) In $\triangle$ABE and $\triangle$ACF,

$\angle$ A= $\angle$ A [Common]

$\angle$ AEB = $\angle$ AFC = 90° [Given]

BE = CF [Given]

$\therefore$ $\triangle$ABE $\cong$ $\triangle$ ACF [By ASA congruency]

(ii) Since $\triangle$ABE $\cong$ $\triangle$ACF

$\Rightarrow$ BE = CF [By C.P.C.T.]

$\Rightarrow$ ABC is an isosceles triangle.

11. ABC and DBC are two isosceles triangles on the same base BC (See figure). Show that $\angle$ ABD = $\angle$ ACD.

Ans. In isosceles triangle ABC,

AB = AC [Given]

$\angle$ ACB = $\angle$ ABC ....(i) [Angles opposite to equal sides]

Also in Isosceles triangle BCD.

BD = DC

$\therefore$ $\angle$ BCD = $\angle$ CBD ....(ii) [Angles opposite to equal sides]

Adding eq. (i) and (ii),

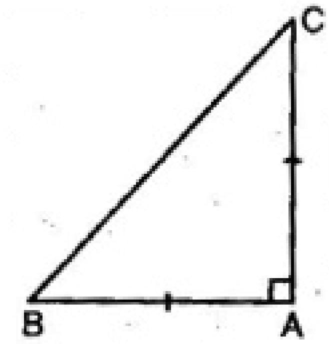
$\angle$ ACB + $\angle$ BCD = $\angle$ ABC + $\angle$ CBD

$\Rightarrow$ $\angle$ ACD = $\angle$ ABD

Or $\angle$ ABD = $\angle$ ACD

12. ABC is a right angled triangle in which $\angle$ A = 90° and AB = AC. Find $\angle$ B and $\angle$ C.

Ans. ABC is a right triangle in which,



$\angle$ A = 90° And AB = AC

In $\triangle$ABC,

AB = AC $\Rightarrow$ $\angle$ C = $\angle$ B ....(i)

We know that, in $\triangle$ABC, $\angle$ A + $\angle$ B + $\angle$ C = 180° [Angle sum property]

$\Rightarrow$ $90^\circ + \angle$ B + $\angle$ B = 180° [$\angle$ A = 90° (given) and $\angle$ B = $\angle$ C (from eq. (i)]

$\Rightarrow$ 2 $\angle$ B = 90°

$\Rightarrow \angle B = 45^\circ$

Also $\angle C = 45^\circ$ [$\angle B = \angle C$]

13. AD is an altitude of an isosceles triangle ABC in which AB = AC. Show that:

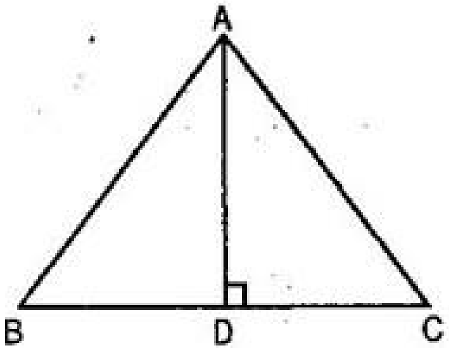
(i) AD bisects BC.

(ii) AD bisects $\angle$ A.

Ans. In $\triangle$ABD and $\triangle$ACD,

AB = AC [Given]

$\angle$ADB = $\angle$ADC = 90° [AD $\perp$ BC]



AD = AD [Common]

$\therefore$ $\triangle$ABD $\cong$ $\triangle$ACD [RHS rule of congruency]

$\Rightarrow$ BD = DC [By C.P.C.T.]

$\Rightarrow$ AD bisects BC

Also $\angle$BAD = $\angle$CAD [By C.P.C.T.]

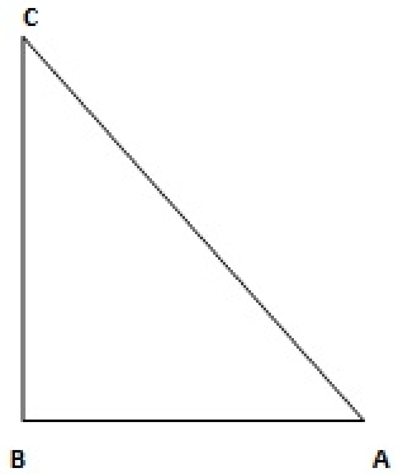
$\Rightarrow$ AD bisects $\angle$A.

14. Show that in a right angles triangle, the hypotenuse is the longest side.

Ans. Given: Let ABC be a right angled triangle, right angled at B.

To prove: Hypotenuse AC is the longest side.

Proof: In right angled triangle ABC,



$\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow \angle A + 90^\circ + \angle C = 180^\circ$ [$\because \angle B = 90^\circ$]

$\Rightarrow \angle A + \angle C = 180^\circ - 90^\circ$

And $\angle B = 90^\circ$

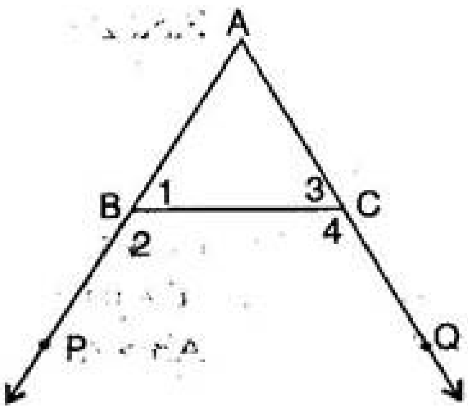
$\Rightarrow \angle B > \angle C$ and $\angle B > \angle A$

Since the greater angle has a longer side opposite to it.

$\Rightarrow$ AC > AB and AC > AB

Therefore $\angle$B being the greatest angle has the longest opposite side AC, i.e. hypotenuse.

15. In figure, sides AB and AC of $\triangle$ABC are extended to points P and Q respectively. Also $\angle$PBC< $\angle$QCB. Show that AC > AB.



Ans. Given: In $\triangle$ABC, $\angle$PBC< $\angle$QCB

To prove: AC > AB

Proof: In $\triangle$ABC,

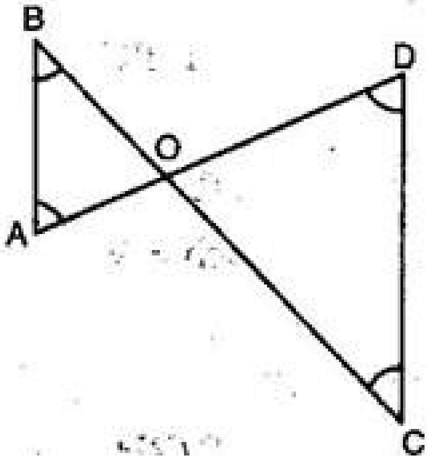
$\angle 4 > \angle 2$ [Given]

Now $\angle 1 + \angle 2 = \angle 3 + \angle 4 = 180^\circ$ [Linear pair]

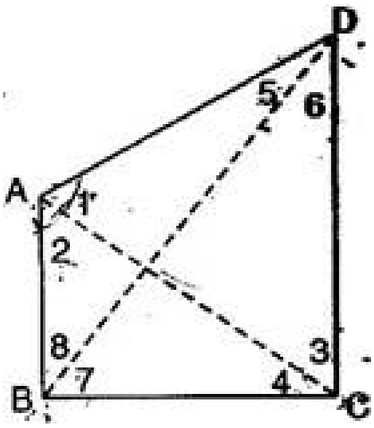
$\therefore \angle 1 > \angle 3$ [$\because \angle 4 > \angle 2$]

$\Rightarrow$ AC > AB [Side opposite to greater angle is longer]

16. In figure, $\angle$B < $\angle$A and $\angle$C < $\angle$D. Show that AD < BC.



Ans. In $\triangle$AOB,



$\angle$B < $\angle$A [Given]

$\Rightarrow$ OA < OB ....(i) [Side opposite to greater angle is longer]

In $\triangle$COD,

$\angle$C < $\angle$D [Given]

$\Rightarrow$ OD < OC ....(ii) [Side opposite to greater angle is longer]

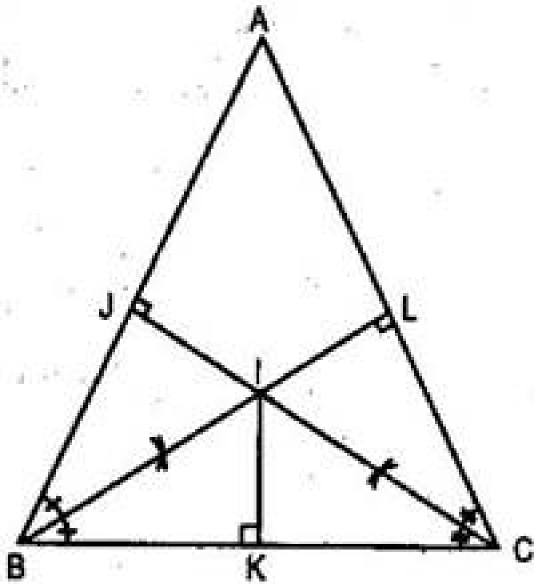
Adding eq. (i) and (ii),

OA + OD < OB + OC

$\Rightarrow$ AD < BC

17. In a triangle locate a point in its interior which is equidistant from all the sides of the triangle.

Ans. Let ABC be a triangle.



Draw bisectors of $\angle$ B and $\angle$ C.

Let these angle bisectors intersect each other at point I.

Draw IK $\perp$ BC

Also draw IJ $\perp$ AB and IL $\perp$ AC.

Join AI.

In $\triangle$BIK and $\triangle$BIJ,

$\angle$ IKB = $\angle$ IJB = 90° [By construction]

$\angle$ IBK = $\angle$ IBJ

[∵ BI is the bisector of ∠B (By construction)]

BI = BI [Common]

∴ △BIK ≅ △BIJ [ASA criteria of congruency]

∴ IK = IJ [By C.P.C.T.] ....(i)

Similarly, △CIK ≅ △CIL

∴ IK = IL [By C.P.C.T.] ....(ii)

From eq (i) and (ii),

IK = IJ = IL

Hence, I is the point of intersection of angle bisectors of any two angles of △ABC equidistant from its sides.

18. In quadrilateral ACBD, AB=AD and AC bisects ∠A. show △ABC ≅ △ACD?

Ans. IN △ ABC and △ACD,

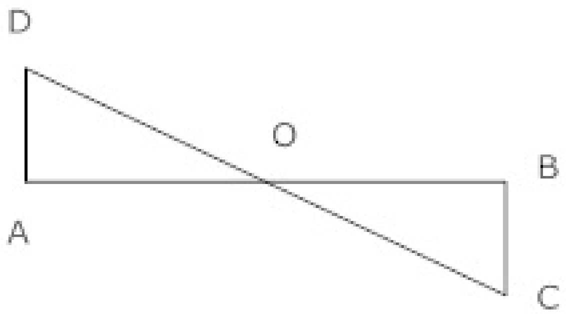
AD=AB......... (Given)

∠BAC= ∠CAD....... (AC bisects ∠A)

And AC= AC .... (Common)

∴ △ABC ≅ △ACD .... (SAS axiom)

19. If DA and CB are equal perpendiculars to a line segment AB. Show that CD bisects AB.



Ans. In $\triangle$AOD and $\triangle$BOC,

AD=BC .... (Given)

$\angle A = \angle B$......... (Each 90°)

And $\angle AOD \cong \angle BOC$ (vert opp. Angles)

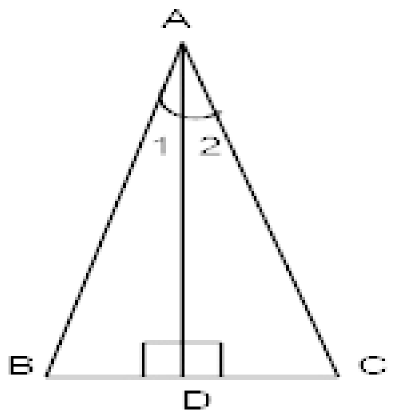
$\therefore \Delta AOD \cong \Delta BOC$ (AAS rule)

$\therefore OA = OB$ (CPCT)

Hence CD bisects AB.

20. *l* and *m*, two parallel lines, are intersected by Another pair of parallel lines *p* and C. show that $\triangle$ABC $\cong$ $\triangle$CDA.

Ans. $L \parallel M$ and AC cuts them – (Given)



$\therefore \angle ACB = \angle CAD$ (alternate angles)

$P \prod Q$ and AC cuts them (Given)

$\therefore \angle CAB = \angle ACD$ (Alternate angles)

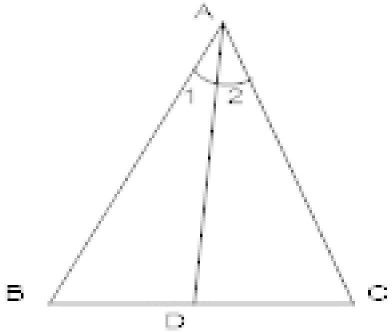
AC=CA (common)

$\therefore \Delta ABC \cong \Delta CDA$ (ASA rule)

21. In fig, the bisector AD of $\triangle$ABC is $\perp$ to the opposite side BC at D. show that $\triangle$ABC is

isosceles?

Ans. In $\triangle$ABD and $\triangle$ACD



$\angle 1 = \angle 2$...... (AD is the bisector of $\angle$A)

And $\angle$ADB= $\angle$ADC=90°..........($AD \perp BC$)

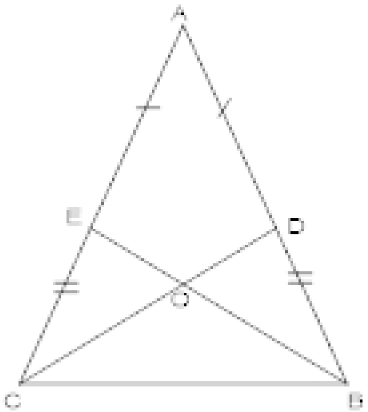
$\therefore AD = AD$........($common$)

$\Delta ABD \cong \Delta ACD$ ...... (ASA rule)

$\therefore$ AB=AC .... (C.P.C.T)

Hence Δ ABC is isosceles.

22. If AE=AD and BD=CE. Prove that $\triangle$AEB $\cong$ $\triangle$ADC



Ans. We have,

AE=AD and CE=BD

$\Rightarrow$ AE+CE=AD+BD

$\Rightarrow$AC=AB(i)

Now, in $\triangle$AEB and $\triangle$ADC,

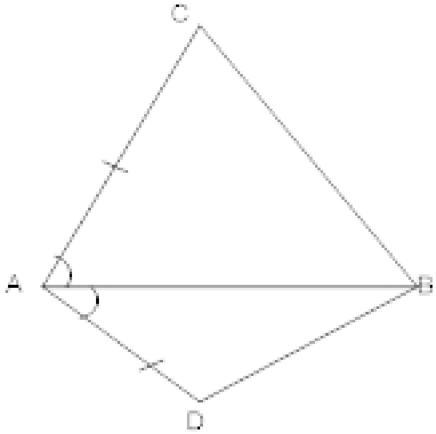
AE=AD [given]

$\angle$ EAB= $\angle$ DAC [common]

AB=AC [from (i)]

$\triangle AEB \cong \triangle ADC$ [by SAS]

23. In quadrilateral ACBD, AC=AD and AB bisects $\angle$ A. show that $\triangle ABC \cong \triangle ABD$. What can you say about BC and BD?



Ans. In $\triangle$ABC and $\triangle$ABD,

AC=AD [given]

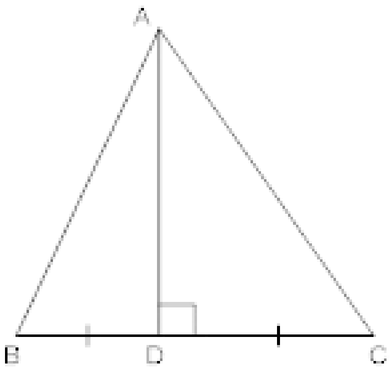
$\angle$CAB=$\angle$DAB[AB bisects $\angle$A]

AB=AB [common]

$\triangle ABC \cong \triangle ABD$[SAS criterion]

$\therefore$ BC=BD [CPCT]

24. In $\triangle$ABC, the median AD is $\perp$ to BC. Prove that $\triangle$ABC is an isosceles triangle.



Ans. *In Δs ABD and ACD,*

BD =CD [D is mid-point of BC]

AD=AD [Common]

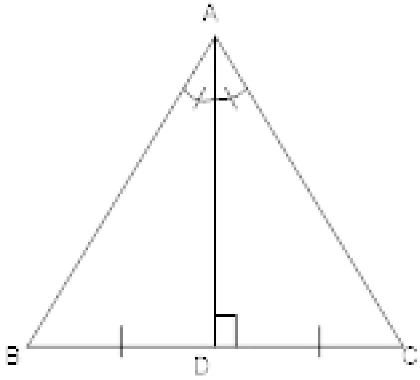
$\angle ADB = \angle ADC$ [each 90^0, $\because AD \perp BC$]

$\Delta ABD \cong \Delta ACD$ [By SAS]

$\therefore AB = AC$ [CPCT]

Hence, triangle ABC is an isosceles triangle.

25. Prove that $\triangle$ABC is isosceles if altitude AD bisects $\angle$ BAC.



Ans. $In\ \Delta s\ ABD\ and\ ACD,$

$\angle ADB = \angle ADC$ [$Each\ 90^0, AD \perp BC$]

$\angle BAD = \angle CAD$ [AD bisects $\angle BAC$]

AD=AD [common]

$\Delta ABD \cong \Delta ACD$ [By AAs]

$\Rightarrow AB = AC$ [CPCT]

Thus, ΔABC is an isosceles triangle.

26. ABC is An isosceles triangle in which altitudes BE and CF are drawn to side AC and AB respectively. Show that these altitudes are equals.

Ans. $In\ \Delta ABE\ and \Delta ACF,$

$\angle A = \angle A$ [common]

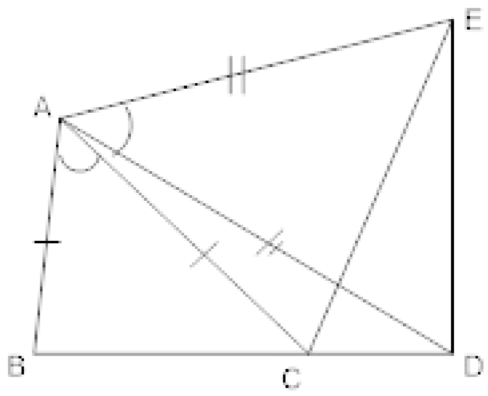
$\angle AEB = \angle AFC = 90^0$

AB=AC [given]

$\therefore\ \Delta ABE \cong \Delta ACF$ [AAS rule]

$\Rightarrow BE = CF$ [CPCT]

27. If AC= AE, AB=AD and $\angle BAD = \angle EAC$. show that BC =DE.



Ans. $In\ \Delta\ BAC\ and\ \Delta DAE,$

AB=AD [given]

AC=AE [given]

Also, $\angle BAD = \angle EAC$ [given]

$\therefore\ \angle BAC + \angle DAC = \angle EAC + \angle CAD$

$\Rightarrow \angle BAC = \angle EAD$

$\therefore\ \ \Delta\ BAC \cong \Delta\ DAE$ [SAS criterior]

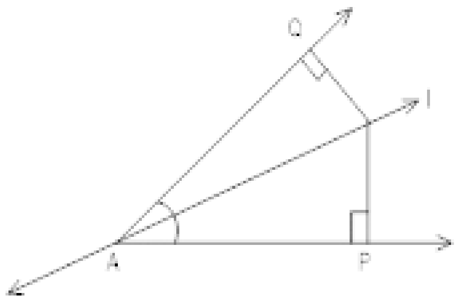
$\Rightarrow BC = DE$ [CPCT]

28. Line $\angle$ is the bisector of an angle $\angle$ A and B is any point on line l. BP and BQ are $\perp$ from B to the arms of $\angle$ A show that :

(i) Δ APB $\cong$ Δ AQB

(ii) BP = BQ or B is A equidistant from the arms of $\angle A$

Ans. $In\ \Delta\ ABP\ and\ \Delta ABQ,$



$\angle BAP = \angle BAQ$ [given]

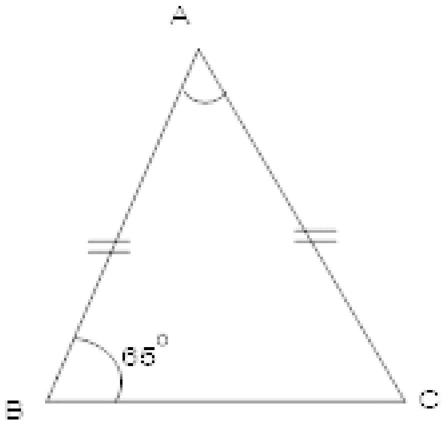
$\angle APB = \angle AQB = 90^0$ [common]

AB=AB [Common]

$(i) \therefore \ \Delta ABP \cong \Delta ABQ$ [AAS rule]

$(ii) BP = BQ$ [CPCT]

29. In the given figure, ΔABC is an isosceles triangle and $\angle$ B = 75°, find x.



Ans. $In\ \Delta s\ ABC,$

AB=AC

$\Rightarrow \angle B = \angle C$ [Angles opposite to equal sides are equal]

But $\angle B = 75^0$

$\therefore \qquad \angle B = \angle C = 75^0$

So,

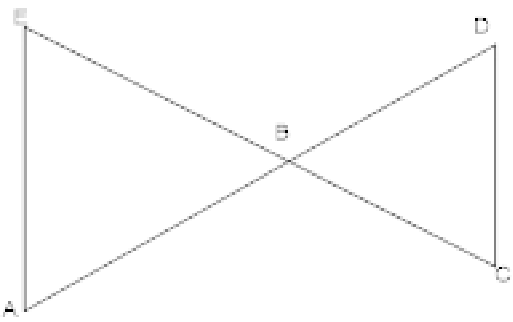
$\angle A + \angle B + \angle C = 180^0$

$x + 150 = 180^0$

$x = 30^0$

30. If $\angle$ E> $\angle$ A and $\angle$ C> $\angle$ D. prove that AD>EC.

Ans. $In\ \Delta\ ABE,$



$\angle E > \angle A$ [given]

$\Rightarrow AB > EB$ [Side opposite to greater angle is larger] .....(i)

Similarly, in $\Delta\ BCD,$

$\angle C > \angle D$ [Given]

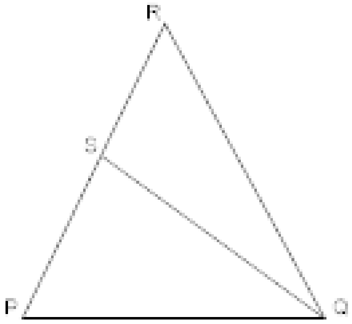
$\Rightarrow BD > BC \quad \rightarrow (ii)$

Adding (i) and (ii)

$AB + BD > EB + BC$

$Or\ AD > EC$

31. If PQ= PR and S is any point on side PR. Prove that RS<QS.



Ans. $In\ \Delta\ PQR,$

PQ=PR [given]

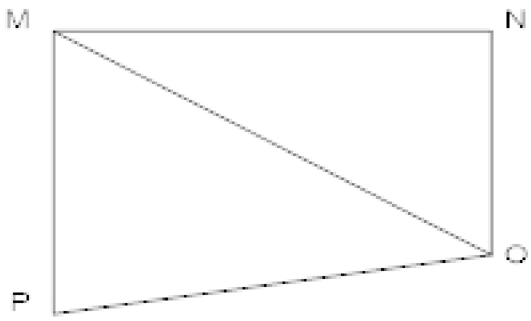
$\Rightarrow \angle PRQ = \angle PQR$ [angle opposite to equal side are equal]

Now, $\angle SQR < \angle PQR$ $[\angle SQR\ is\ a\ part\ of\ \angle PQR]$

$\therefore \angle SQR < \angle PRQ\ OR\ \angle SRQ$

$\Rightarrow RS < QS$ [side opposite to smaller angle in ΔSRQ]

32. Prove that MN+NO +OP+PM>2MO.



Ans. $In\ \Delta\ MON,$

MN+NO>MO [Sum of any two side of Δ is greater than third sides] ...(i)

Similarly in Δ MPQ,

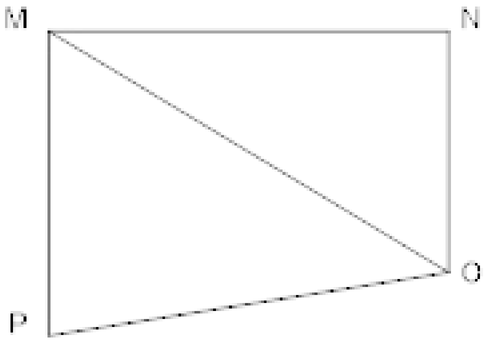
OP+PM>MO ....(ii)

Hence from (i) and (ii)

Or MN+NO+OP+PM>2MO

33. Prove that MN+NO+OP>PM.

Ans. $In\ \Delta\ MON,$



MN+NO>MO [Sum of any two side of Δ is greater than third sides] ...(i)

Similarly in Δ MOQ,

MO+OP>PM ....(ii)

Hence from (i) and (ii)

Or MN+NO+OP+MO>MO+PM

Or MN+NO+OP>PM

34. Δ ABC is an equilateral triangle and $\angle B = 60^0$, find $\angle C$.

Ans. $In\ \Delta\ ABC,$

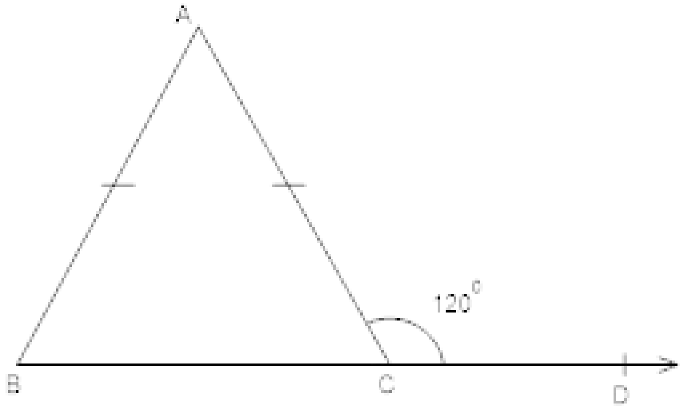
AB=AC

$\Rightarrow \angle B = \angle C$ [angles opposite to equal sides are equal]

But $\angle B = 60^0$

So, $\angle C = 60^0$

35. In the figure, AB = AC and $\angle ACD = 120^0$, $find \angle B$.



Ans. $Since$ in Δ ABC, AB = AC

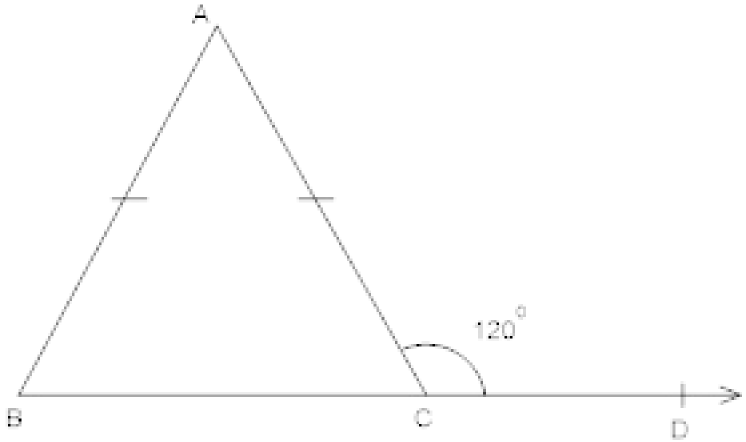
$\Rightarrow \angle B = \angle C$ [angles opposite to equal sides are equal]

Also, $\angle ACB + \angle ACD = 180^0$ [Linear pair]

$\Rightarrow \angle ACB = 180^0 - 120^0$

and, $\angle C = \angle B = 60^0$

36. In the given figure, find $\angle A$



Ans. $In\ \Delta$ ABC,

$\angle A + \angle B + \angle C = 180^0$ [sum of three angles of a]

$\angle A + 60^0 + 60^0 = 180^0$

$\Rightarrow \angle A = 180^0 - 120^0$

$\angle A = 60^0$

CBSE Class 9 Mathemaics

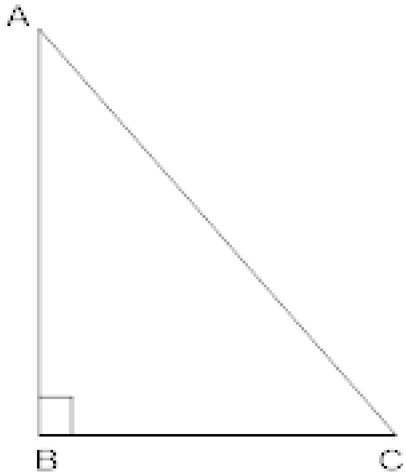
Important Questions

Chapter 7

Triangles

3 Marks Quetions

1. Prove that in a right triangle, hypotenuse is the longest (or largest) side.



Ans. Given a right angled triangle ABC in which $\angle B = 90^0$

$\therefore$ AC is its hypotenuse.

Now, since

$\angle B = 90^0$

$\therefore A + \angle B + \angle C = 180^0$

$\angle A + \angle C = 180^0 - 90^0 = 90^0$

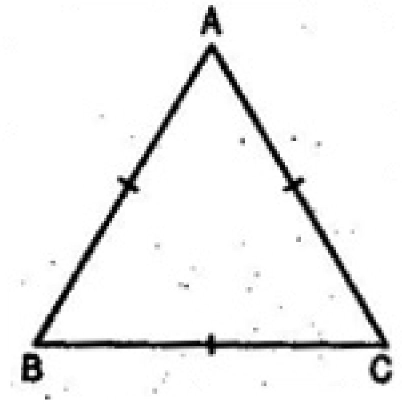
i.e. $\angle B = \angle A + \angle C$

$\Rightarrow \angle B > \angle A$ and $\angle B > \angle C$

Hence, the side opposite to $\angle B$ is the hypotenuse and the longest side of the triangle.

2. Show that the angles of an equilateral triangle are 60^{o} each.

Ans. Let ABC be an equilateral triangle.



$\therefore AB = BC = AC \Rightarrow AB = BC$

$\Rightarrow \angle C = \angle A$..........(i)

Similarly, AB = AC

$\Rightarrow \angle C = \angle B$..........(ii)

From eq. (i) and (ii),

$\angle A = \angle B = \angle C$..........(iii)

Now in $\triangle$ ABC

$\angle A + \angle B + \angle C = 180^{\circ}$ ....(iv)

$\Rightarrow \angle A + \angle A + \angle A = 180^{\circ} \Rightarrow 3\angle A = 180^{\circ}$

$\Rightarrow \angle A = 60^{\circ}$

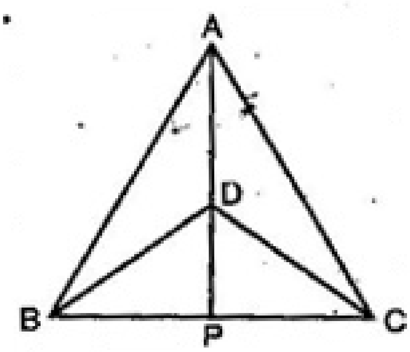
Since $\angle A = \angle B = \angle C$[From eq. (iii)]

$\therefore \angle A = \angle B = \angle C = 60^{\circ}$

Hence each angle of equilateral triangle is 60°.

3. $\triangle$ ABC and $\triangle$ DBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (See figure). If AD is extended to intersect BC at P, show

that:



(i) △ ABD ≅ △ ACD

(ii) △ ABP ≅ △ ACP

(iii) AP bisects ∠A as well as ∠D.

(iv) AP is the perpendicular bisector of BC.

Ans. i) △ ABC is an isosceles triangle.

∴ AB = AC

△ DBC is an isosceles triangle.

∴ BD = CD

Now in △ ABD and △ ACD,

AB = AC[Given]

BD = CD[Given]

AD = AD[Common]

∴ △ ABD ≅ △ ACD[By SSS congruency]

⇒ ∠BAD = ∠CAD[By C.P.C.T.]..........(i)

(ii) Now in △ ABP and △ ACP,

AB = AC[Given]

∠BAD = ∠CAD[From eq. (i)]

AP = AP

$\therefore \triangle ABP \cong \triangle ACP$[By SAS congruency]

(iii) Since $\triangle ABP \cong \triangle ACP$[From part (ii)]

$\Rightarrow \angle BAP = \angle CAP$[By C.P.C.T.]

$\Rightarrow$ AP bisects $\angle A$.

Since $\triangle ABD \cong \triangle ACD$[From part (i)]

$\Rightarrow \angle ADB = \angle ADC$[By C.P.C.T.]..........(ii)

Now $\angle ADB + \angle BDP = 180^\circ$ [Linear pair]..........(iii)

And $\angle ADC + \angle CDP = 180^\circ$ [Linear pair]..........(iv)

From eq. (iii) and (iv),

$\angle ADB + \angle BDP = \angle ADC + \angle CDP$

$\Rightarrow \angle ADB + \angle BDP = \angle ADB + \angle CDP$[Using (ii)]

$\Rightarrow \angle BDP = \angle CDP$

$\Rightarrow$ DP bisects $\angle D$orAP bisects $\angle D$.

(iv) Since $\triangle ABP \cong \triangle ACP$[From part (ii)]

$\therefore$ BP = PC[By C.P.C.T.]..........(v)

And $\angle APB = \angle APC$[By C.P.C.T.]..........(vi)

Now $\angle APB + \angle APC = 180^\circ$ [Linear pair]

$\Rightarrow \angle APB + \angle APC = 180^\circ$ [Using eq. (vi)]

$\Rightarrow 2\angle APB = 180^\circ$

$\Rightarrow \angle APB = 90^\circ$

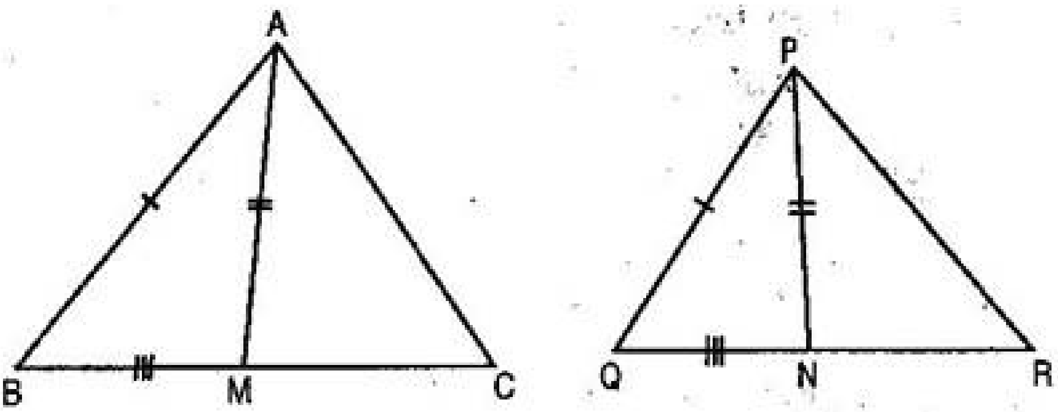
$\Rightarrow$ AP $\perp$ BC..........(vii)

From eq. (v), we have BP PC and from (vii), we have proved AP $\perp$ B. So, collectively AP is perpendicular bisector of BC.

4. Two sides AB and BC and median AM of the triangle ABC are respectively equal to side PQ and QR and median PN of $\triangle$ PQR (See figure). Show that:

(i) $\triangle$ ABM $\cong$ $\triangle$ PQN

(ii) $\triangle$ ABC $\cong$ $\triangle$ PQR



Ans. AM is the median of $\triangle$ ABC.

$\therefore$ BM = MC = $\frac{1}{2}$ BC..........(i)

PN is the median of $\triangle$ PQR.

$\therefore$ QN = NR = $\frac{1}{2}$ QR..........(ii)

Now BC = QR[Given] $\Rightarrow$ $\frac{1}{2}$ BC = $\frac{1}{2}$ QR

$\therefore$ BM = QN..........(iii)

(i) Now in $\triangle$ ABM and $\triangle$ PQN,

AB = PQ[Given]

AM = PN[Given]

BM = QN[From eq. (iii)]

$\therefore \triangle$ ABM $\cong \triangle$ PQN[By SSS congruency]

$\Rightarrow \angle$ B = $\angle$ Q[By C.P.C.T.]..........(iv)

(ii) In $\triangle$ ABC and $\triangle$ PQR,

AB = PQ[Given]

$\angle$ B = $\angle$ Q[Prove above]

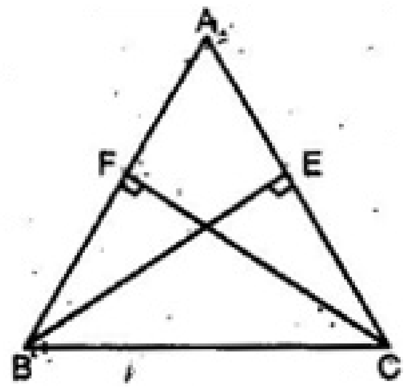
BC = QR[Given]

$\therefore \triangle$ ABC $\cong \triangle$ PQR[By SAS congruency]

5. BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.

Ans. In $\triangle$ BEC and $\triangle$ CFB,

$\angle$ BEC = $\angle$ CFB[Each 90°]



BC = BC[Common]

BE = CF[Given]

$\therefore \triangle$ BEC $\cong \triangle$ CFB[RHS congruency]

$\Rightarrow$ EC = FB[By C.P.C.T.].....(i)

Now In $\triangle$ AEB and $\triangle$ AFC

$\angle$ AEB = $\angle$ AFC [Each 90°]

$\angle$ A = $\angle$ A[Common]

BE = CF[Given]

$\therefore$ $\triangle$ AEB $\cong$ $\triangle$ AFC[ASA congruency]

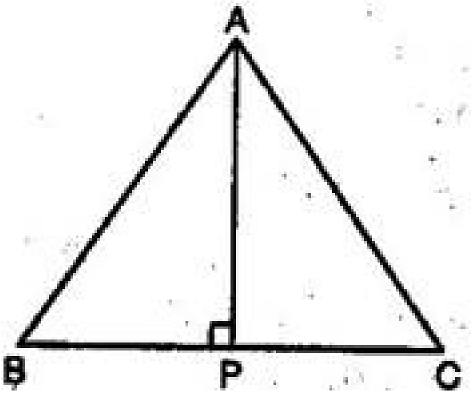
$\Rightarrow$ AE = AF[By C.P.C.T.]............(ii)

Adding eq. (i) and (ii), we get,

EC + AE = FB + AF $\Rightarrow$ AB = AC

$\Rightarrow$ ABC is an isosceles triangle.

6. ABC is an isosceles triangles with AB = AC. Draw AP $\perp$ BC and show that $\angle$ B = $\angle$ C.



Ans. Given: ABC is an isosceles triangle in which AB = AC

To prove: $\angle$ B = $\angle$ C

Construction: Draw AP $\perp$ BC

Proof: In $\triangle$ ABP and $\triangle$ ACP

$\angle$ APB = $\angle$ APC = 90° [By construction]

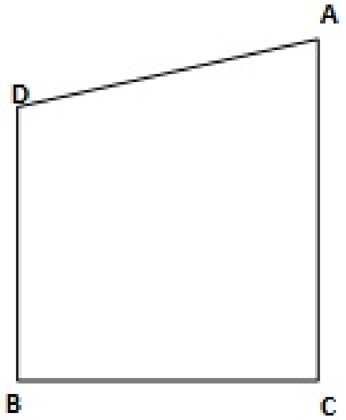
AB = AC[Given]

AP = AP[Common]

∴ △ ABP ≅ △ ACP[RHS congruency]

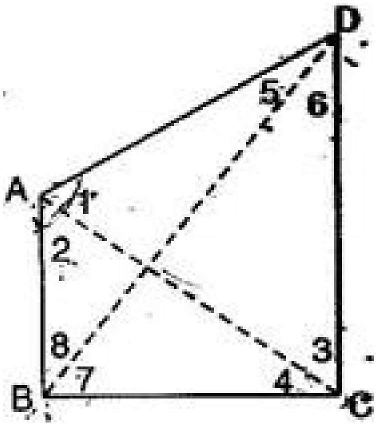
⇒ ∠ B = ∠ C[By C.P.C.T.]

7. AB and CD are respectively the smallest and longest sides of a quadrilateral ABCD (See figure). Show that ∠ A > ∠ C and ∠ B > ∠ D.



Ans. Given: ABCD is a quadrilateral with AB as smallest and CD as longest side.

To prove:(i) ∠ A > ∠ C(ii) ∠ B > ∠ D



Construction: Join AC and BD.

Proof:(i)In △ ABC, AB is the smallest side.

∴ ∠ 4 < ∠ 2 ....(i)

[Angle opposite to smaller side is smaller]

In △ ADC, DC is the longest side.

∴ ∠ 3 < ∠ 1 ....(ii)

[Angle opposite to longer side is longer]

Adding eq. (i) and (ii),

$\angle 4 + \angle 3 < \angle 1 + \angle 2 \Rightarrow \angle C < \angle A$

(ii) In $\triangle$ ABD, AB is the smallest side.

$\therefore \angle 5 < \angle 8$ ....(iii)

[Angle opposite to smaller side is smaller]

In $\triangle$ BDC, DC is the longest side.

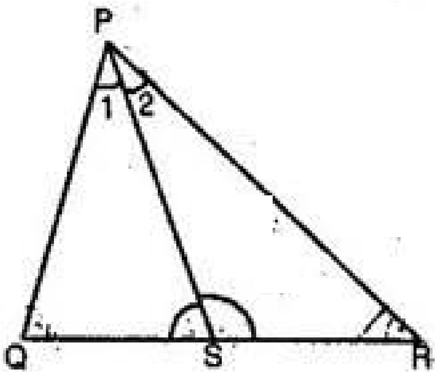
$\therefore \angle 6 < \angle 7$ ....(iv)

[Angle opposite to longer side is longer]

Adding eq. (iii) and (iv),

$\angle 5 + \angle 6 < \angle 7 + \angle 8 \Rightarrow \angle D < \angle B$

8. In figure, PR > PQ and PS bisects $\angle$QPR. Prove that $\angle$PSR> $\angle$PSQ.



Ans. In $\triangle$ PQR,PR > PQ[Given]

$\therefore \angle$PQR> $\angle$PRQ.....(i)[Angle opposite to longer side is greater]

Again $\angle 1 = \angle 2$.....(ii)[$\because$ PS is the bisector of $\angle$P]

$\therefore \angle PQR + \angle 1 > \angle PRQ + \angle 2$..........(iii)

But $\angle PQS + \angle 1 + \angle PSQ = \angle PRS + \angle 2 + \angle PSR = 180^\circ$ [Angle sum property]

$\Rightarrow \angle PQR + \angle 1 + \angle PSQ = \angle PRQ + \angle 2 + \angle PSR$.........(iv)

[$\angle PRS = \angle PRQ$ and $\angle PQS = \angle PQR$]

From eq. (iii) and (iv),

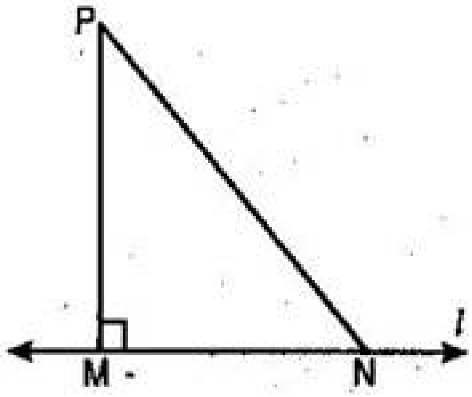
$\angle PSQ < \angle PSR$

Or $\angle PSR > \angle PSQ$

9. Show that all the line segments drawn from a given point not on it, the perpendicular line segment is the shortest.

Ans. Given: l is a line and P is point not lying on l. PM $\perp l$

N is any point on l other than M.



To prove: PM <PN

Proof: In $\triangle$PMN, $\angle$M is the right angle.

$\therefore$N is an acute angle. (Angle sum property of $\triangle$)

$\therefore \angle M > \angle N$

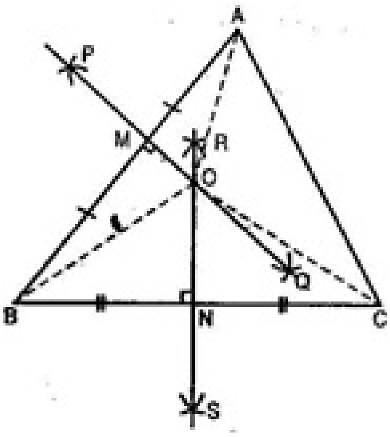
$\therefore$PN> PM[Side opposite greater angle]

$\Rightarrow$ PM <PN

Hence of all line segments drawn from a given point not on it, the perpendicular is the shortest.

10. ABC is a triangle. Locate a point in the interior of $\triangle$ ABC which is equidistant from all the vertices of $\triangle$ ABC.

Ans. Let ABC be a triangle.



Draw perpendicular bisectors PQ and RS of sides AB and BC respectively of triangle ABC. Let PQ bisects AB at M and RS bisects BC at point N.

Let PQ and RS intersect at point O.

Join OA, OB and OC.

Now in $\triangle$ AOM and $\triangle$ BOM,

AM = MB[By construction]

$\angle$ AMO = $\angle$ BMO = 90° [By construction]

OM = OM[Common]

$\therefore$ $\triangle$ AOM $\cong$ $\triangle$ BOM[By SAS congruency]

$\Rightarrow$ OA = OB[By C.P.C.T.].....(i)

Similarly $\triangle$ BON $\cong$ $\triangle$ CON

$\Rightarrow$ OB = OC[By C.P.C.T.].....(ii)

From eq. (i) and (ii),

OA = OB = OC

Hence O, the point of intersection of perpendicular bisectors of any two sides of $\triangle$ ABC equidistant from its vertices.

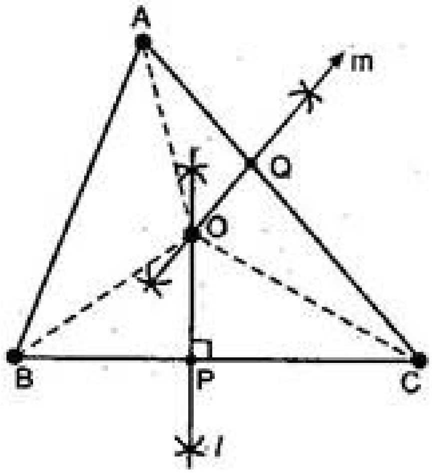
11. In a huge park, people are concentrated at three points (See figure).

A: where there are different slides and swings for children.

B: near which a man-made lake is situated.

C: which is near to a large parking and exit.

Where should an ice cream parlour be set up so that maximum number of persons can approach it?



Ans. The parlour should be equidistant from A, B and C.

For this let we draw perpendicular bisector say l of line joining points B and C also draw perpendicular bisector say m of line joining points A and C.

Let l and m intersect each other at point O.

Now point O is equidistant from points A, B and C.

Join OA, OB and OC.

Proof: In $\triangle$ BOP and $\triangle$ COP,

OP = OP[Common]

$\angle$ OPB = $\angle$ OPC = 90°

BP = PC[P is the mid-point of BC]

$\therefore \triangle$ BOP $\cong \triangle$ COP[By SAS congruency]

$\Rightarrow$ OB = OC[By C.P.C.T.].....(i)

Similarly, $\triangle$ AOQ $\cong \triangle$ COQ

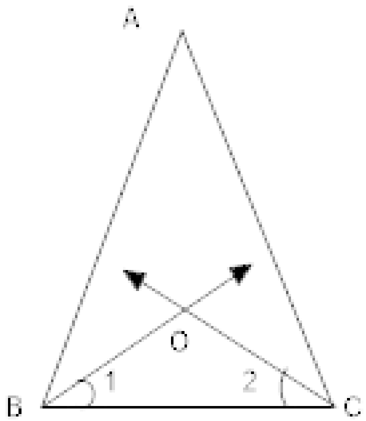
$\Rightarrow$ OA = OC[By C.P.C.T.].....(ii)

From eq. (i) and (ii),

OA = OB = OC

Therefore, ice cream parlour should be set up at point O, the point of intersection of perpendicular bisectors of any two sides out of three formed by joining these points.

12. If $\triangle$ABC, the bisector of $\angle$ ABC and $\angle$ BCA intersect each other at the point O prove that $\angle$ BOC = $90^\circ + \frac{1}{2}\angle A$.



Ans. *In* Δ BOC, we have

$\angle 1+\angle 2+\angle BOC = 180^\circ \rightarrow (1)$

In ΔABC, *we have*

$\angle A+\angle B+\angle C=180^\circ$

$\Rightarrow \angle A + 2(\angle 1) + 2(\angle 2) = 180^\circ$

$\Rightarrow \frac{\angle A}{2} + \angle 1 + \angle 2 = 90^\circ$

$\Rightarrow \angle 1 + \angle 2 = 90 - \frac{\angle A}{2}$

Substituting this value of $\angle 1 + \angle 2$ in (1)

$90^\circ - \frac{\angle A}{2} + \angle BOC = 180^\circ$

$\angle BOC = 90^\circ + \frac{\angle A}{2}$

So, $\angle BOC = 90^\circ + \frac{\angle A}{2}$

13. Prove that if one angle of a triangle is equal to the sum of the other two angles, the triangle is right angled:

Ans. $\angle A + \angle B + \angle C = 180^\circ$ Sum of three angles of triangle is 180°] .....(1)

Given that: $\angle A + \angle C = \angle B \rightarrow (2)$

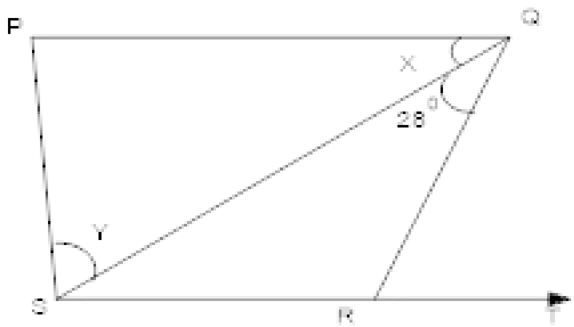
From (1) and (2)

$\angle B + \angle B = 180^\circ$

$\Rightarrow \angle B = \frac{180^\circ}{2} = 90^\circ$

Hence ΔABC is right angled.

14. IF fig, if PQ $\perp$ PS, PQ || SR, $\angle$ SQR = 28° and $\angle$ QRT = 65°, then find the values of X and Y.



Ans. $PQ \| SR$ and QR is the transversal,

$\therefore \angle PQR = \angle QRT$ [pair of alternate angles]

Or $\angle PQS + \angle SQR = \angle QRT$

or $x + 28^\circ = 65^\circ$

$\therefore x = 65^\circ - 28^\circ = 37^\circ$

Also in Δ PQS,

$\angle SPQ + \angle PSQ + \angle PQS = 180^\circ$

$\Rightarrow 90^\circ + y + x = 180^\circ$

Or $90^\circ + y + 37^\circ = 180^\circ$

$y = 53^\circ$

15. If in fig, AD= AE and D and E are point on BC such that BD=EC prove that AB=AC.

Ans. $In\ \Delta\ ADE,$

$AD = AE$ [Given]

$\therefore \angle ADE = \angle AED$ [angles opposite to equal side are equal]

Now, $\angle ADE + \angle ADB = 180^\circ$ [linear pair]

Also, $\angle AED + \angle AEC = 180^\circ$ [linear pair]

$\Rightarrow \angle ADE + \angle ADB = \angle AED + \angle AEC$

But, $\angle ADE = \angle AED$

Now in, Δ ABD and Δ ACE,

BD=CE

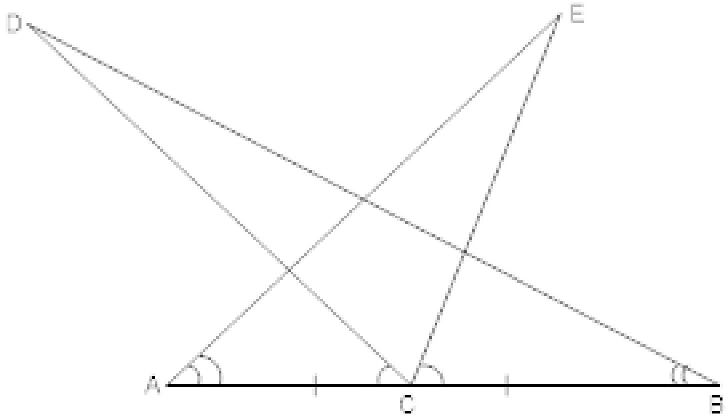
AD=AE

$\angle ADB = \angle AEC$

$\therefore$ $\Delta ABC \cong \Delta ACE$ [By SAS]

$\Rightarrow AB = AC$ [CPCT]

16. In the given figure, AC=BC, $\angle$DCA=$\angle$ECB and $\angle$DBC=$\angle$EAC. Prove that $\triangle$DBC and $\triangle$EAC are congruent and hence DC=EC.



Ans. We have,

$\angle DCA = \angle ECB$ [Given]

$\Rightarrow \angle DCA + \angle ECD = \angle ECB + \angle ECD$ [adding $\angle$ECD on both sides]

$\Rightarrow \angle ECA = \angle DCB$ ...(i)

$\angle DCB = \angle ECA$ [From (i)]

Now, in Δs DBC and EAC

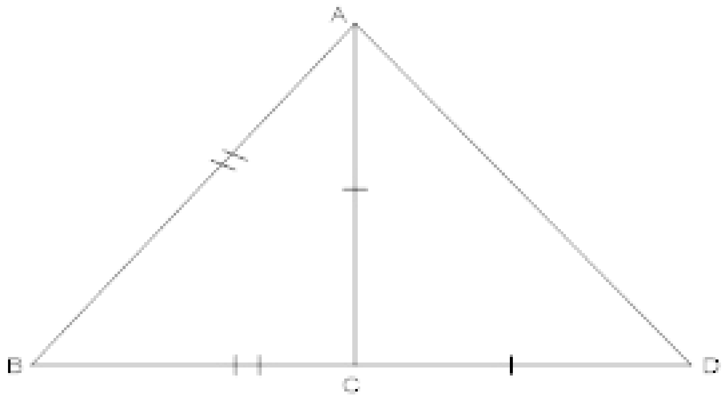
$BC = AC$ [given]

$\angle DBC = \angle EAC$ [given]

$\Delta DBC \cong \Delta EAC$ [By SAS]

$\Rightarrow DC = EC$ [CPCT]

17. From the following figure, prove that $\angle$ BAD=3 $\angle$ ADB.



Ans. $Let \angle ADC = Q$

$\Rightarrow \angle CAD = Q \ [\because CA = CD]$

Exterior $\angle ACB = \angle CAD = Q + Q$ = 2Q

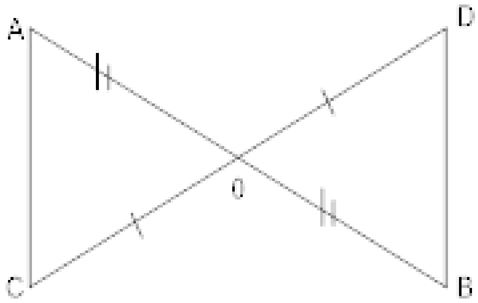
$\Rightarrow \angle BAC = 2Q \ [\because BA = BC]$

$\angle BAD = \angle BAC + \angle CAD$

Hence $= 2Q + Q$

$= 3Q = 3\angle ADC = 3\angle ADB$

18. O is the mid-point of AB and CD. Prove that AC=BD and AC $\parallel$ BD.



Ans. $In \ \Delta s \ AOC \ and \ BOD$

$AO = OB$ [O is the mid – point of AB]

$\angle AOC = \angle BOD$ [vertically opposite angles]

$CO = OD$ [O is the mid-point of CD]

$\Delta AOC \cong \Delta BOD$ [By SAS]

$AC = BD$ [CPCT]

$\Rightarrow \angle CAO = \angle DBO$ [CPCT]

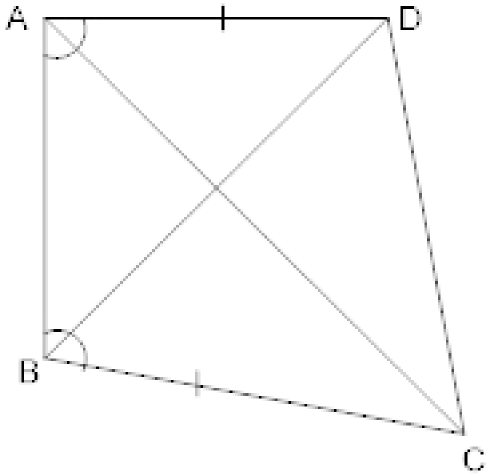
Now, AC and BD are two lines inter sected by a transversal AB such that $\angle CAO = \angle DBO$ i.e. alternate angle are equal.

19. ABCD is a quadrilateral in which AD=BC and $\angle$DAB=$\angle$CBA. Prove that.

(i) ΔABD $\cong$ ΔBAC

(ii) BA=AC

(iii) $\angle$ ABD=$\angle$ BAC



Ans. $In\ \Delta s\ ABD\ and\ BAC,$

$AD = BC$ [given]

$\angle DAB = \angle CBA$ [given]

$AB = AB$ [common]

$(i)\ \because\ \Delta ABD \cong \Delta BAC$ [SAS criterion]

$(ii) \Rightarrow \therefore BD = AC$ [CPCT]

$(iii) \Rightarrow Also \angle ABD = \angle BAC$ [CPCT]

20. AB is a line segment. AX and BY are equal two equal line segments drawn on opposite side of line AB such that AX $\parallel$ BY. If AB and XY intersect each other at P. prove that

(i) $\triangle$APX $\cong$ $\triangle$ BPY,

(ii) AB and XY bisect each other at P.

Ans. $In\ \Delta\ APX\ and\ \Delta BPY,$

$\angle 1 = \angle 2$ [alternate angle]

$\angle 3 = \angle 4$ [vertically opposite angle]

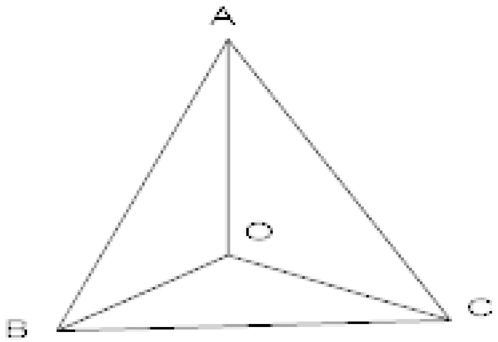
AX=BY[given]

$\therefore \Delta APX \cong \Delta BPY$ [By AAS]

$\Rightarrow AP = BP\ and\ PX = PY$ [CPCT]

$\Rightarrow$ AB and XY bisects each other at P.

21. In an isosceles $\triangle$ABC, with AB =AC, the bisector of $\angle$B and $\angle$C intersect each other at o, join A to o. show that:



(i) OB=OC

(ii) AO bisects $\angle$A.

Ans. (i) In Δ ABC,

AB=AC [given]

$\angle$ ACB=$\angle$ABC [angles opposite to equal side]

$\therefore \frac{1}{2} \angle ACB = \frac{1}{2} \angle ABC$

or $\angle OCB = \angle OBC$

$\Rightarrow$ OB=OC [side opposite to equal angle]

(ii) In ΔAOB and Δ AOC

AB = AC [given]

$\angle$ ABO=$\angle$ACO [Halves of equals]

OB=OC [proved]

$\therefore \Delta AOB \cong \Delta AOC$ [SAS rule]

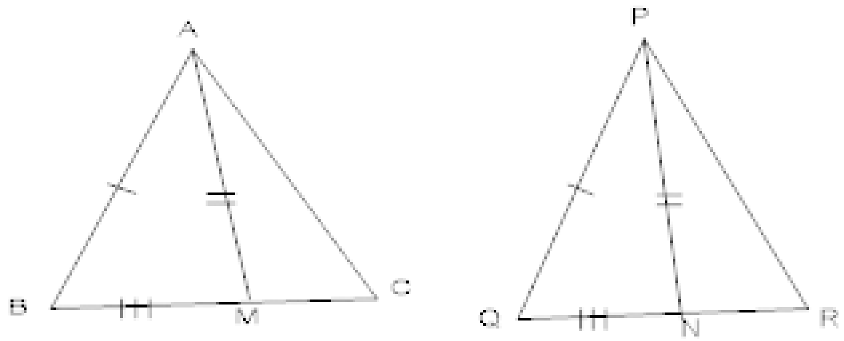
$\Rightarrow \angle BAO = \angle CAO$ [CPCT]

i.e. AO bisects $\angle A$

22. Two side AB and BC and median AM of a triangle ABC are respectively equal to side PQ and QR and median PN of ΔPQR, show that

(i) $\Delta ABM \cong \Delta PQN$

(ii) $\Delta ABC \cong \Delta PQR$



Ans. (i) In ΔABM and ΔPQN,

AB=PQ [Given]

BM=QN [Halves of equal]

AP=PN[Given]

$\therefore \Delta ABM \cong \Delta PQN$ [SSS rules]

(ii) $\Rightarrow$ $\angle B = \angle Q$

Now, in Δs ABC and PQR,

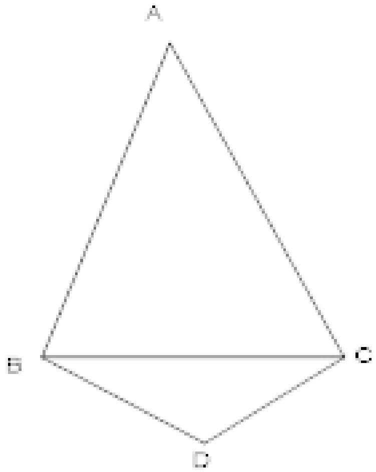
AB=PQ [Given]

BC=QR [Given]

$\angle B = \angle Q$ [Proved]

$\therefore \Delta ABC \cong \Delta PQR$ [SAS rule]

23. In the given figure, ABC and DBC are two triangles on the same base BC such that AB=AC and DB=DC. Prove that $\angle$ ABD = $\angle$ ACD,



Ans. $In\ \Delta ABC,$

AB=AC[Given]

$\therefore \angle ABC = \angle ACB$ [angles opposite to equal side are equals]

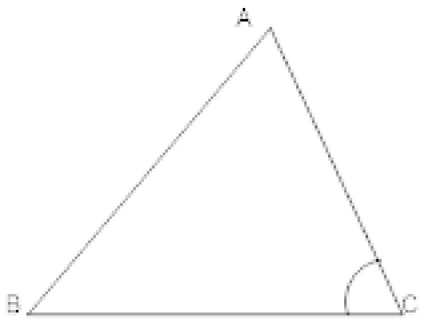
Similarly in, ΔDBC, $DB = DC$ [Given].....(1)

$\therefore \angle DBC = \angle DCB$ ......(2)

Adding (1) and (2)

$\angle ABC + \angle DBC = \angle ACB + \angle DCB$
or $\angle ABD = \angle ACD$

24. Prove that the Angle opposite to the greatest side of a triangle is greater than two-third of a right angle i.e. greater than 60°



Ans. $In\ \Delta\ ABC,$

$AB > BC$ [Given]

$\angle C > \angle A$ [angle opposite to large side is greater]....(i)

Similarly,

AB>AC

$\therefore\ \angle C > \angle B \rightarrow$ (ii)

Adding (i) and (ii)

$2\angle C > (\angle A + \angle B)$

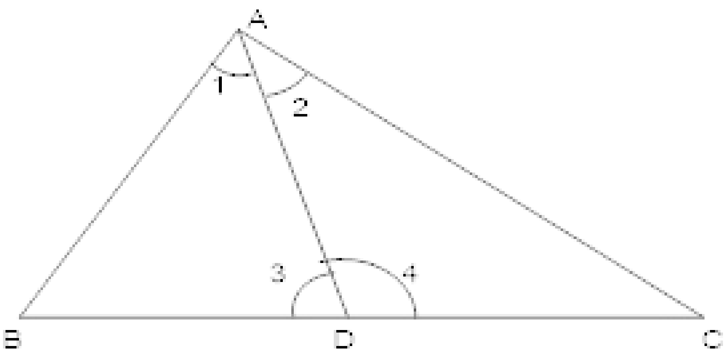
Adding $\angle C$ to both sides,

$3\angle C > (\angle A + \angle B + \angle C)$

$3\angle C > 180^0$ [Sum of three angles of Δ is 180^0]

$Or,\ \angle C > 60^0$

25. AD is the bisector of $\angle$ A of ΔABC, where D lies on BC. Prove that AB>BD and AC>CD.



Ans. $In\ \Delta\ ADC,$

$\angle 3 > \angle 2$ [Exterior angles of Δ is greater than each of the interior opposite angles]

But $\angle 2 = \angle 1$ [Ad bisects $\angle A$]

$\therefore\ \angle 3 = \angle 1$ [Side opposite to greater angle is larger]

$\Rightarrow$ AB > BD

In Δ ABD,

$\angle 4 > \angle 1$ [Exterior angles of Δ is greater than each of the interior opposite angles]

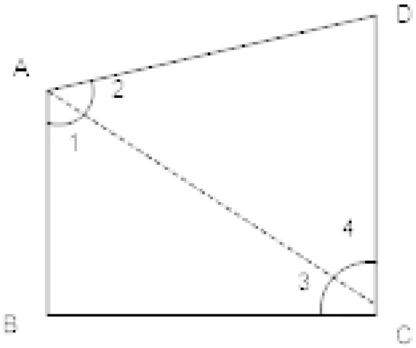
But, $\angle 1 = \angle 2$

$\therefore \angle 4 > \angle 2$

$\Rightarrow$ AC > CD

[Side opposite to greater angle is larger].

26. In the given figure, AB and CD are respectively the smallest and the largest side of a quadrilateral ABCD. Prove that $\angle$A> $\angle$C and $\angle$B> $\angle$D.



Ans. Join AC.

In Δ ABC,

BC > AB [AB is the smallest sides of quadrilateral ABCD]

$\Rightarrow \angle 1 > \angle 3$ [Angle opposite to larger side is greater]...(i)

In Δ ADC,

CD > AD [CD is the largest side of quadrilateral ABCD]

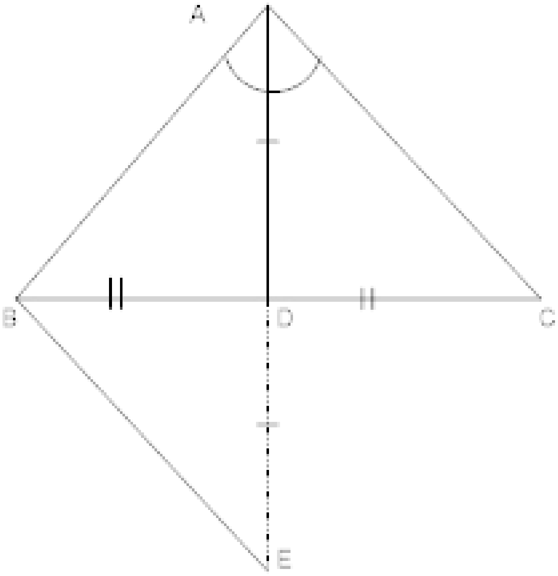
$\angle 2 > \angle 4$ [angle opposite to larger side is greater].....(ii)

Adding (i) and (ii)

$\angle 1 + \angle 2 > \angle 3 + \angle 4$ Or $\angle A > \angle C$

Similarly, by joining BD, we can show that $\angle B > \angle D$

27. If the bisector of a vertical angle of a triangle also bisects the opposite side; prove that the triangle is an isosceles triangle.



Ans. $In\ \Delta ADC$ and ΔEDB,

DC=DB [Given]

AD=ED [By construction]

$\angle ADC = \angle EDB$ [vertically opposite angle]

$\therefore \Delta ADC \cong \angle EDB$ [By SAS]

$\Rightarrow AC = EB$ and

$\angle DAC = \angle DEB$ [CPCT]

But, $\angle DAC = \angle BAD$ [$\because$ AD bisects $\angle A$]

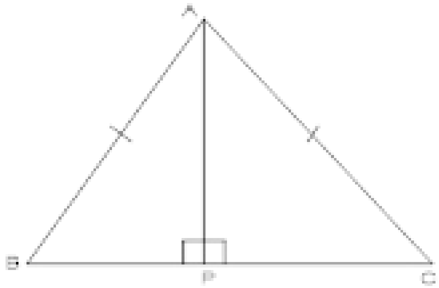
$\therefore\ \angle BAD = \angle DEB$

$\Rightarrow AB = BE$

But BE=AC [Proved above]

$\therefore\ AB = AC$

28. ABC is an isosceles triangle with AB = AC. Draw AP $\perp$ BC to show that $\angle B = \angle C$.



Ans. *In right* ΔAPB and ΔAPC,

AP=AP [common]

Hypotenuse AB = Hypotenuse AC [Given]

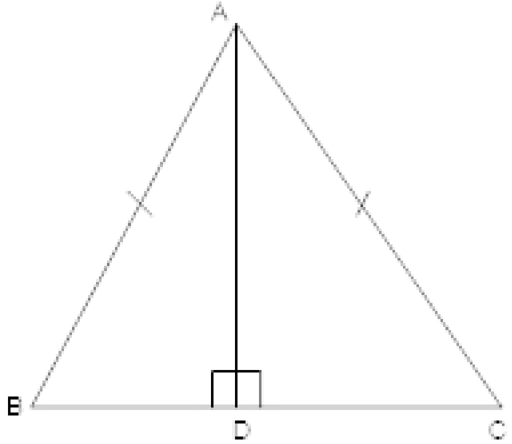
$\therefore \Delta\ APB \cong \Delta\ APC$ [RHS rule]

$\Rightarrow \angle B = \angle C$ [CPCT]

29. AD is an altitude of an isosceles triangle ABC in which AB = AC. Prove that:

(i) AD bisects BC

(ii) AD bisects $\angle A$



Ans. (i) In right triangle ABD and ACD,

Side AD = Side AD[common]

Hypotenuse AB = Hypotenuse AC [Given]

$\therefore \Delta\ ABD \cong \Delta\ ACD$ [By RSH]

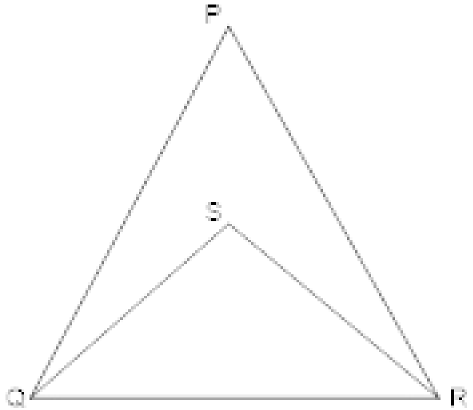
$\Rightarrow BD = CD$ [CPCT]

Also, AD bisects BC

(ii) Also, $\angle BAD = \angle CAD$ [CPCT]

i.e. AD bisects $\angle A$.

30. In the given figure, PQ>PR, QS and RS are the bisectors of the $\angle$Q and $\angle$R respectively. Prove that SQ>SR.



Ans. Since PQ>PR

$\therefore \angle R > \angle Q$ [angle opposite to larger side is larger]

$$\Rightarrow \frac{1}{2}\angle R > \frac{1}{2}\angle Q$$

$\Rightarrow \angle SRQ > \angle SQR$

$\Rightarrow SQ > SR$ [Side opposite to greater angle is larger]

CBSE Class 9 Mathemaics

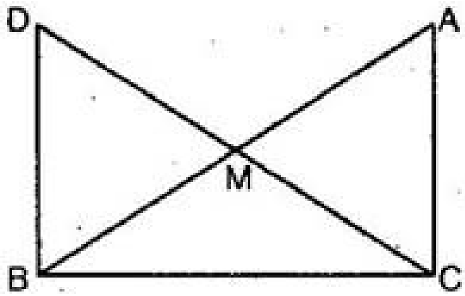
Important Questions

Chapter 7

Triangles

4 Marks Quetions

1. In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B. (See figure)



Show that:

(i) $\triangle$ AMC $\cong$ $\triangle$ BMD

(ii) $\angle$ DBC is a right angle.

(iii) $\triangle$ DBC $\cong$ $\triangle$ ACB

(iv) CM = $\frac{1}{2}$ AB

Ans. (i) In $\triangle$ AMC and $\triangle$ BMD,

AM = BM [AB is the mid-point of AB]

$\angle$ AMC = $\angle$ BMD [Vertically opposite angles]

CM = DM [Given]

$\therefore$ $\triangle$ AMC $\cong$ $\triangle$ BMD [By SAS congruency]

$\therefore$ $\angle$ ACM = $\angle$ BDM ....(i)

$\angle$ CAM = $\angle$ DBM and AC = BD [By C.P.C.T.]

(ii) For two lines AC and DB and transversal DC, we have,

$\angle$ACD = $\angle$BDC [Alternate angles]

$\therefore$ AC $\parallel$ DB

Now for parallel lines AC and DB and for transversal BC.

$\angle$DBC = $\angle$ACB [Alternate angles] ....(ii)

But $\triangle$ABC is a right angled triangle, right angled at C.

$\therefore$ $\angle$ACB = 90° ....(iii)

Therefore $\angle$DBC = 90° [Using eq. (ii) and (iii)]

$\Rightarrow$ $\angle$DBC is a right angle.

(iii) Now in $\triangle$DBC and $\triangle$ABC,

DB = AC [Proved in part (i)]

$\angle$DBC = $\angle$ACB = 90° [Proved in part (ii)]

BC = BC [Common]

$\therefore$ $\triangle$DBC $\cong$ $\triangle$ACB [By SAS congruency]

(iv) Since $\triangle$DBC $\cong$ $\triangle$ACB [Proved above]

$\therefore$ DC = AB

$\Rightarrow$ AM + CM = AB

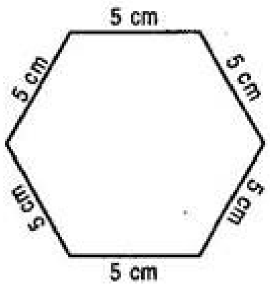
$\Rightarrow$ CM + CM = AB [$\because$DM = CM]

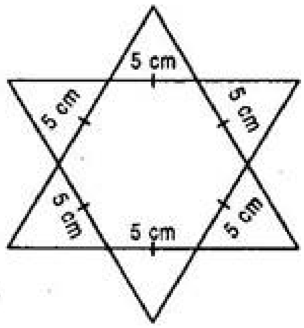
$\Rightarrow$ 2CM = AB

$\Rightarrow$ CM = $\frac{1}{2}$ AB

2. Complete the hexagonal rangoli and the star rangolies (See figure) but filling them with as many equilateral triangles of side 1 cm as you can. Count the number of

triangles in each case. Which has more triangles?





Ans. In hexagonal rangoli, Number of equilateral triangles each of side 5 cm are 6.

Area of equilateral triangle = $\frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}}{4}(5)^2 = \frac{\sqrt{3}}{4}\times 25$ sq. cm

Area of hexagonal rangoli = 6 $\times$ Area of an equilateral triangle

$= 6 \times \frac{\sqrt{3}}{4}\times 25 = 150\times\frac{\sqrt{3}}{4}$ sq. cm ....(i)

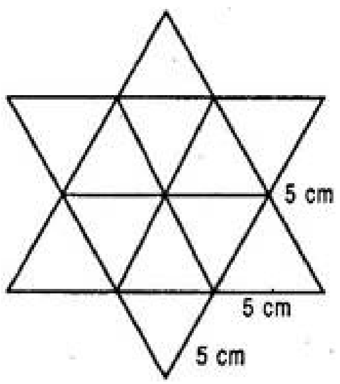
Now area of equilateral triangle of side 1 cm = = $\frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}}{4}(1)^2 = \frac{\sqrt{3}}{4}$ sq. cm .....(ii)

Number of equilateral triangles each of side 1 cm in hexagonal rangoli

$= 150\times\frac{\sqrt{3}}{4} \div \frac{\sqrt{3}}{4} = 150\times\frac{\sqrt{3}}{4}\times\frac{4}{\sqrt{3}}$ = 150 .....(iii)

Now in Star rangoli,

Number of equilateral triangles each of side 5 cm = 12



Therefore, total area of star rangoli = 12 $\times$ Area of an equilateral triangle of side 5 cm

$= 12 \times \left(\frac{\sqrt{3}}{4}(5)^2 \right)$

$= 12 \times \frac{\sqrt{3}}{4} \times 25$

$= 300 \frac{\sqrt{3}}{4}$ sq. cm ....(iv)

Number of equilateral triangles each of side 1 cm in star rangoli

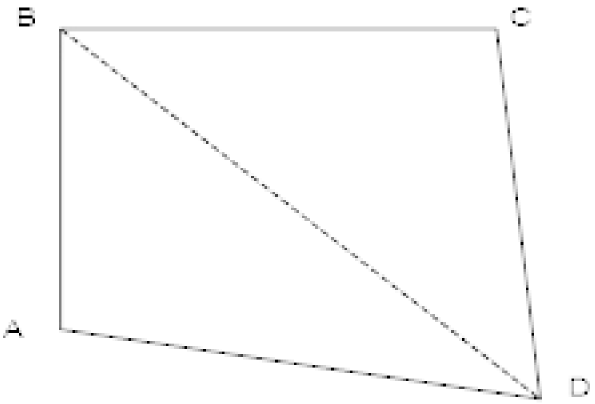
$= 300 \frac{\sqrt{3}}{4} \div \frac{\sqrt{3}}{4}$

$= 300 \frac{\sqrt{3}}{4} \times \frac{4}{\sqrt{3}}$

= 300 ....(v)

From eq. (iii) and (v), we observe that star rangoli has more equilateral triangles each of side 1 cm.

3. Prove that sum of the quadrilateral is 360^0 ?



Ans. Join B and D to obtain two triangles ABD Δ BCD.

$\angle BAD + \angle ABD + \angle BDA = 180^\circ$ [sum of three angles of Δ is 180°] ....(1)

$\angle CBD + \angle BCD + \angle CDB = 180^\circ$ [sum of three angles of Δ is 180°] ....(2)

Adding, (1) and (2)

$\angle BAD+\angle ABD+\angle BDA+\angle CBD+\angle BGCD+\angle BCD+\angle CDB=360^\circ$

Or $\angle BAD+(\angle ABD+\angle CBD)+\angle BCD+(\angle CDB+\angle BDA)=360^\circ$

Or $\angle BAD+\angle ABC+\angle BCD+\angle CDA=360^\circ$

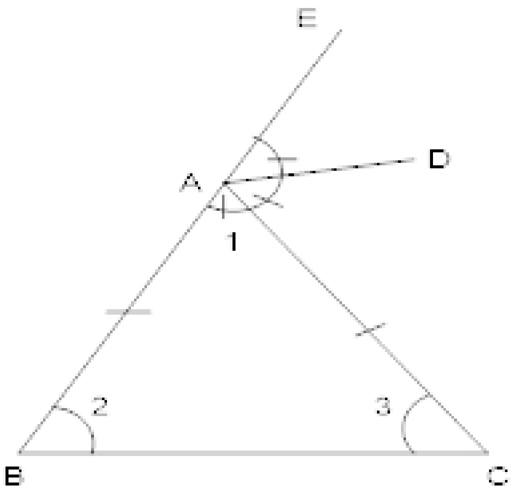
i.e. $\angle A+\angle B+\angle C+\angle D=360^\circ$

So,

Sum of quadrilateral is

Hence proved.

4. Δ ABC is an isosceles triangle with AB=AC. AD bisects the exterior $\angle$ A. prove that AD $\|$ BC.



Ans. Since AD bisects the exterior A,

EAD=$\frac{1}{2}\angle EAC$

$=\frac{1}{2}\left[180^0-\angle 1\right]=90^0-\frac{1}{2}\angle 1$ ...(i)

$\left[\because \angle 1+\angle EAC=180^0 (Linear\ pair)\right]$

$\angle 1+\angle 2+\angle 3=180^0$

$\Rightarrow \angle 1+\angle 2+\angle 2=180^0$ [$\because AB=AC$]

$\Rightarrow 2\angle 2=180^0-\angle 1$

But $\angle 2=90^0-\frac{1}{2}\angle 1$ ...(i)

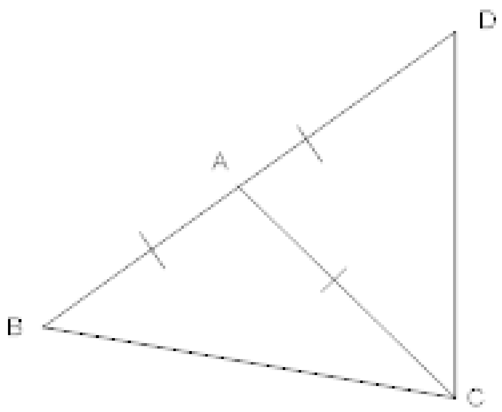
Hence from (i) and (ii)

$\angle EAD=\angle 2=\angle ABC$

But these are corresponding angles

$\therefore AD \parallel BC$

5. Δ ABC is an isosceles triangle in which AB=AC and side BA is produced to D such that AD=AB. Show that $\angle$ BCD is a right angle.



Ans. $\angle ABC=\angle ACB$ [angles opposite to equal side]

Also, $\angle ACD=\angle ADC$ [angles opposite to equal side]

Now $\angle BAC+\angle CAD=180^\circ$ [linear pair]

Also, $\angle CAD=\angle ABC+\angle ACB$ [exterior angle of Δ ABC]

$=2\angle ACB$ [exterior angle of Δ ABC]

Also, $\angle BAC=\angle ACD+\angle ADE$

$= 2\angle ACD$

$\therefore \angle BAC + \angle CAD$

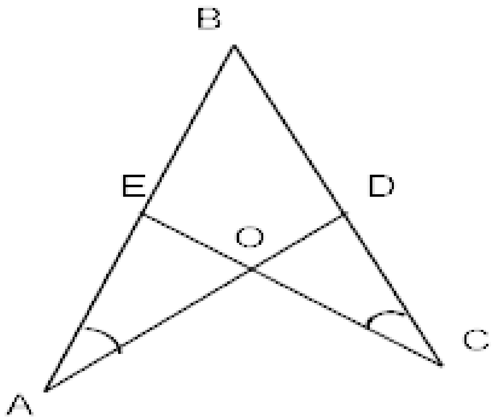
$= 2(\angle ACD + \angle ACB)$

$= 2\angle BCD$

i.e., $2\angle BCD = 180^\circ$

or $\angle BCD = 90^\circ$

6. In the given figure, $\angle A = \angle C$ and AB = BC. Prove that $\triangle ABD \cong \triangle CBE$.



Ans. In Δs AOE and COD,

$\angle A = \angle C$ [Given]

$\angle AOE = \angle COD$ [vertically opposite angle]

$\therefore \angle A + \angle AOE = \angle C + \angle COD$

$\Rightarrow 180^0 - \angle AEO = 180^0 - \angle CDO \left[\because \begin{array}{l} \angle A + \angle AOE + \angle AEO = 180^0 \text{ and} \\ \angle C + \angle COD + \angle CDO = 180^0 \end{array}\right]$

$\Rightarrow \angle AEO = \angle CDO \quad \rightarrow$ (i)

Now, $\angle AEO + \angle OEB = 180^0$ [linear pair]

And $\angle CDO + \angle ODB = 180^0$ [linear pair]

$\Rightarrow \angle AEO + \angle OEB = \angle CDO + \angle ODB$

$\Rightarrow \angle OEB = \angle ODB$ [Using (i)]

$\Rightarrow \angle CEB = \angle ADB \quad \rightarrow$ (ii)

Now, in Δs ABD and CBE,

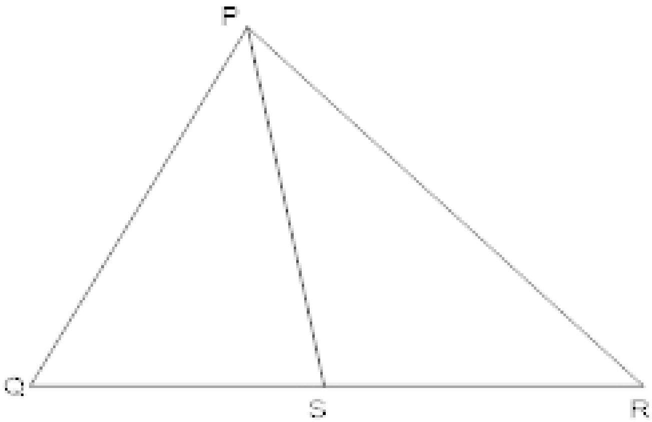
$\angle A = \angle C$ [Given]

$\angle ADB = \angle CEB$ [From (ii)]

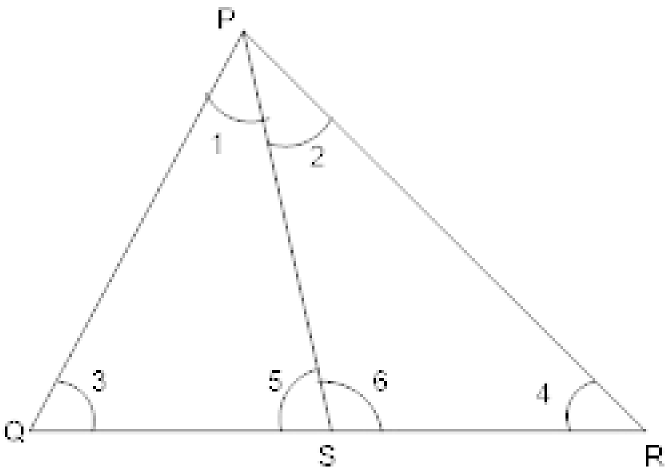
$AB = CB$

$\Delta ABD \cong \Delta CBE$ [By AAS]

7. In the given figure, PR>PQ and PS is the bisector of $\angle QPR$. Prove that $\angle PSR > \angle PSQ$



Ans.



$In\ \Delta$ PQR,

PR > PQ [Given]

$\Rightarrow \angle 3 > \angle 4$ [angle opposite to larger side] ....(i)

Also, $\angle 6 = \angle 1 + \angle 3$ [Exterior angle theorem] ....(ii)

Similarly, $\angle 5 = \angle 2 + \angle 4$

But, $\angle 2 = \angle 1$ [PS bisects $\angle QPR$]

$\therefore \angle 5 = \angle 1 + \angle 4$ .....(iii)

Subtracting (iii) from (ii)

$\angle 6 - \angle 5 = (\angle 1 + \angle 3) - (\angle 1 + \angle 4)$

Or $\angle 6 - \angle 5 = \angle 3 - \angle 4$.....(iv)

Now,

$\angle 3 > \angle 4$

$\Rightarrow \angle 3 - \angle 4 = 0 \quad \rightarrow (v)$

From (iv) and (iii)

$\angle 6 - \angle 5 > 0$
$\angle 6 > \angle 5$

Or $\angle PSR > \angle PSQ$